11. $y = \frac{x}{x^2+1}$

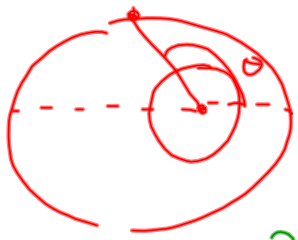
$\frac{dx}{dt} = 3\frac{dy}{dt}$ $\quad \frac{1}{3}\frac{dx}{dt} = \frac{\frac{dx}{dt}(x^2+1) - x(2x)\cdot\frac{dx}{dt}}{(x^2+1)^2}$

23.b. $\theta = 120°$ increasing at 2.7°/min

find the change in A

$r = 3960$ miles

$z = \frac{4995}{1 + .12\cos\theta}$



$\frac{dz}{dt} = \frac{0(1+12\cos\theta) - 4995(.12(-\sin\theta)\frac{d\theta}{dt})}{(1+.12\cos\theta)^2}$

$\frac{dz}{dt} = \frac{4995(.12\sin\theta)\frac{d\theta}{dt}}{(1+.12\cos\theta)^2}$ $\frac{120}{360}\cdot 2\pi = \frac{2\pi}{3}$

$\frac{4995(.12\sin\frac{2\pi}{3})}{(1+.12\cos\frac{2\pi}{3})^2}.047123\frac{2.7°}{360}\cdot 2\pi =$

.2446181 .0471239

$\frac{dz}{dt} = \frac{4995(.12\frac{\sqrt{3}}{2})}{(1-.06)^2}.047123 = 27.68425$ mi/min

$(1-.06)^2 = .88$

$$3\frac{DX}{DT} = \frac{(X^2+1)\frac{DX}{DT} - (2X^2)\frac{DX}{DT}}{(X^2+1)^2}$$

$$(X^2+1)^2 3\frac{DX}{DT} = (X^2+1)\frac{DX}{DT} - (2X^2)\frac{DX}{DT}$$

$$(X^2+1)^2 3 = (1-X^2)$$

$$3X^4+6X^2+3 = 1-X^2$$

$$3x^4+7x^2+2=0 \qquad (x^2+2)(3x^2+1)=0$$

ax^2+bx+c

to factor $3x^2+7x+2 = (x^2+2)(3x^2+1)=0$

① multiply $a \cdot c$
$3 \cdot 2 = 6$

② list all pairs of factors →

③ look at the SIGN of c
(a>0)
Create a column for that →

④ combine the pairs of factors →

② 6 ③ +

$1 \cdot 6$ 7 ④

$2 \cdot 3$ 5

⑤ the quadratic can only be factored if "b" shows up in the column ③ [and the signs work]

multiply = 6

$(1x+2)(3x+1)$

multiply = 1

$$\frac{1}{3}\frac{DX}{DT} = \frac{(X^2+1)\frac{DX}{DT} - (2X^2)\frac{DX}{DT}}{(X^2+1)^2}$$

$$(X^2+1)^2 \cdot \frac{1}{3}\frac{DX}{DT} = (X^2+1)\frac{DX}{DT} - (2X^2)\frac{DX}{DT}$$

$$(X^2+1)^2 = 3(1-X^2)$$

$$X^4+2X^2+1 = 3-3X^2$$

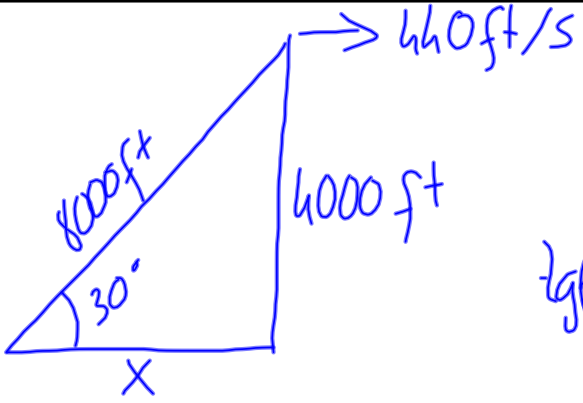
$$X^4+5X^2-2=0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{So } X^2 = \frac{-5\pm\sqrt{(5)^2-4(1)(-2)}}{2(1)} = \frac{-5\pm\sqrt{33}}{2}$$

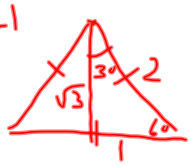
$$\text{so } X = \pm\sqrt{\frac{-5\pm\sqrt{33}}{2}}$$

b



$\frac{d\theta}{dt} = ?$

$\tan\theta = \frac{4000}{x}$: $4000x^{-1}$



$\tan 30 = \frac{1}{\sqrt{3}}$

$x = 4000\sqrt{3}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{-4000\ \frac{dx}{dt}}{x^2}$

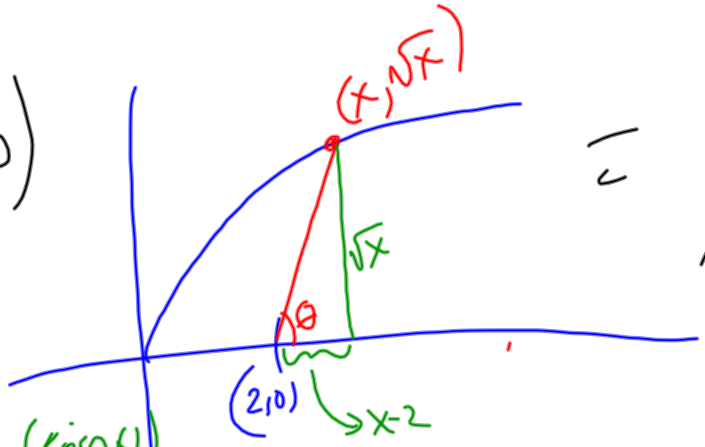
$\frac{1}{\frac{1}{\sqrt{3}}} = \tan 30 = \frac{4000}{x}$

$\frac{1}{\cos(30)} \cdot \frac{d\theta}{dt} = -\frac{4000 \cdot 440}{(4000\sqrt{3})^2}$

$\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} \cdot \frac{d\theta}{dt} = \frac{-4000 \cdot 440}{4000 \cdot 4000 \cdot 3} = \frac{-11}{3 \cdot 100}$

$\frac{1}{\frac{3}{4}} \frac{d\theta}{dt} = \frac{-11}{300}$ so $\frac{d\theta}{dt} = \frac{3}{4} \cdot \frac{-11}{300} = -\frac{11}{400}$ radians/sec $\cdot \frac{180 \text{ deg}}{\pi \text{ rad}}$

4 b)



$y = \sqrt{x}$

$\tan\theta = \frac{\sqrt{3}}{1}$

$\tan\theta = \sqrt{3}$

60

$\tan\theta = \frac{\sqrt{x}}{x-2}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{\frac{1}{2}x^{-\frac{1}{2}}\frac{dx}{dt}(x-2) - \sqrt{x}\frac{dx}{dt}}{(x-2)^2}$

$(x-2)^2 \sec^2(y) \frac{d\theta}{dt} = \frac{dx}{dt}\left(\frac{1}{2}x^{-\frac{1}{2}}(x-2) - \sqrt{x}\right)$

$\frac{3}{4} \quad \frac{3}{1} \quad \frac{3}{1}$

$\sec^2(y) \frac{d\theta}{dt} = 4\left(.288 - \sqrt{3}\right)$

$\sec^2(y) \frac{d\theta}{dt} = 1.1547 - 6.92820$

$\frac{1}{\cos^2 60} .25 \frac{d\theta}{dt} = 1.15$

$\frac{d\theta}{dt} = -23.094$ rad/